International Earth Science Constellation
Mission Operations Working Group
June 13-15, 2017
Earth Observing System Covariance Realism Updates
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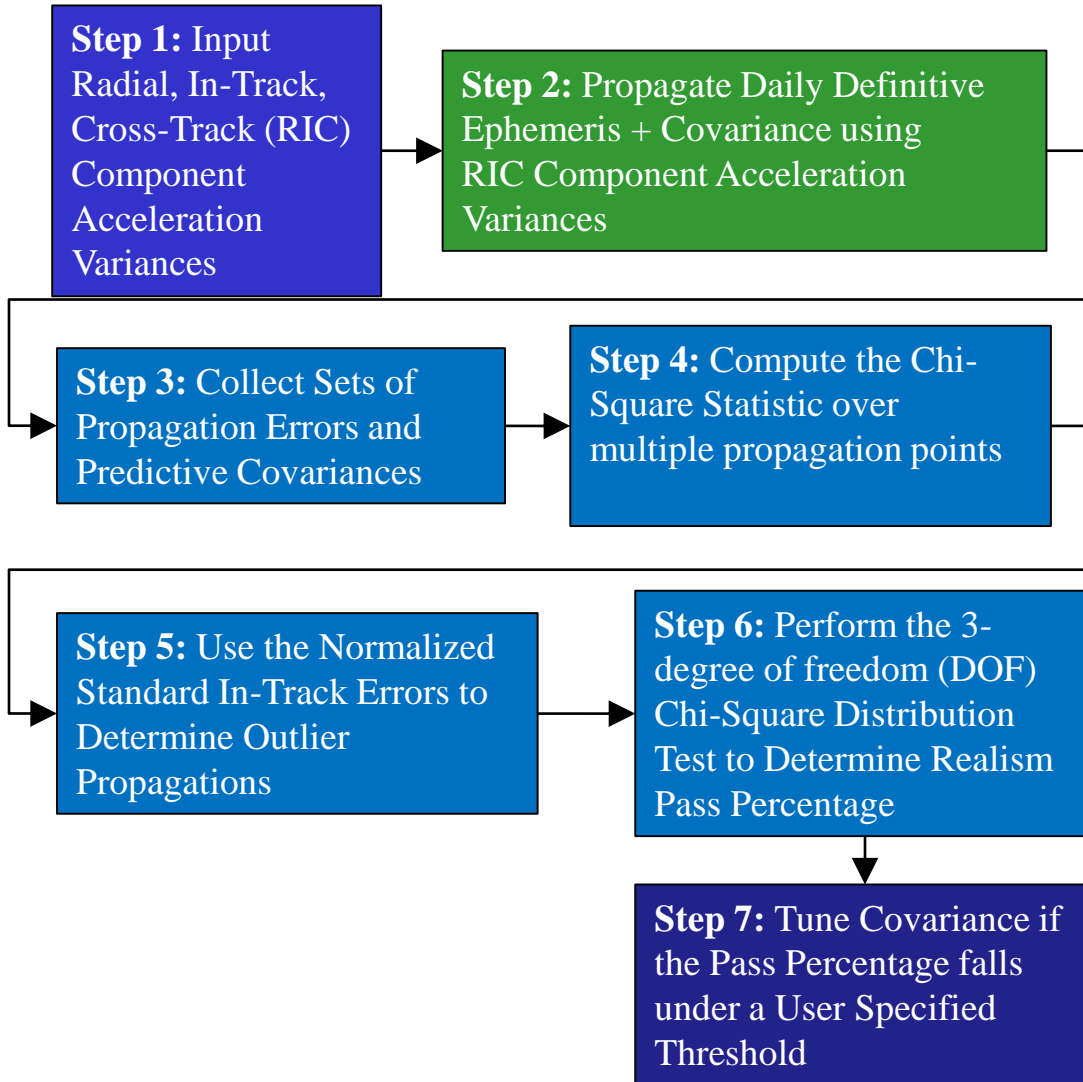EOS FDS, esmo-eos-fds@lists.nasa.gov, +1.301.614.5050

Agenda

- Overview of Aqua/Aura Covariance Operations
  - Earth Observing System (EOS) Flight Dynamics System (FDS) Covariance Realism QA (Quality Assurance) and Tuning Flowchart
  - Covariance QA Automation
  - Aqua and Aura Covariance Tuning
  - Automation Results to Date
  - Covariance Propagation through Maneuvers
- Future Analysis/Work
  - Covariance Propagation Implementation through Maneuvers
  - Covariance Propagation using Polynomial Chaos Expansion
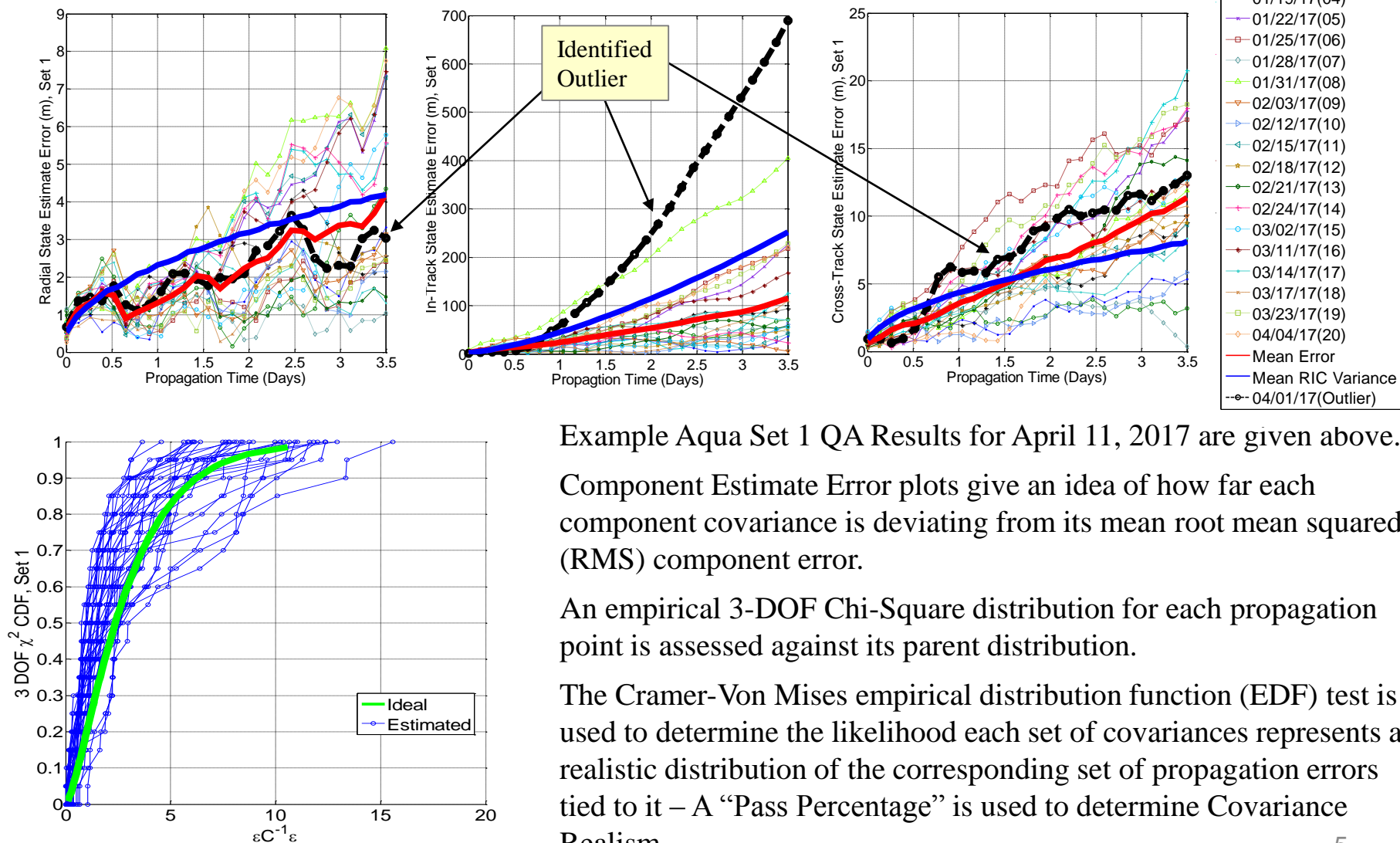- Conclusion

Overview of Aqua/Aura Covariance Operations

- Aqua and Aura Owner/Operator (O/O) covariances are being used in operations to compute the probability of collision (P_C).

- This only includes daily operations and Drag Make-Up (DMU) maneuver planning.

- Software has been delivering tuned covariance since June 14, 2016.

- Software ensures covariances are tuned for periods devoid of persistently high and extreme solar activity as well as post maneuver propagation errors.

- Aqua's last tuning date was on November 7, 2016.

- Aura's last tuning date was on November 9, 2016.

EOS Covariance Realism QA and Tuning Flowchart

Step 1: Input Radial, In-Track, Cross-Track (RIC) Component Acceleration Variances

Step 2: Propagate Daily Definitive Ephemeris + Covariance using RIC Component Acceleration Variances

Step 3: Collect Sets of Propagation Errors and Predictive Covariances

Step 4: Compute the Chi-Square Statistic over multiple propagation points

Step 5: Use the Normalized Standard In-Track Errors to Determine Outlier Propagations

Step 6: Perform the 3-degree of freedom (DOF) Chi-Square Distribution Test to Determine Realism Pass Percentage

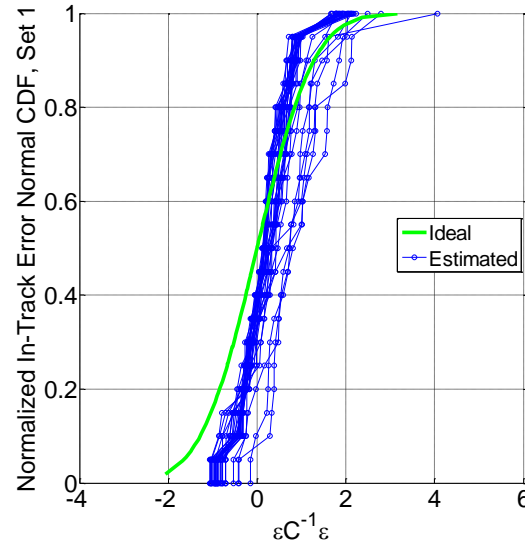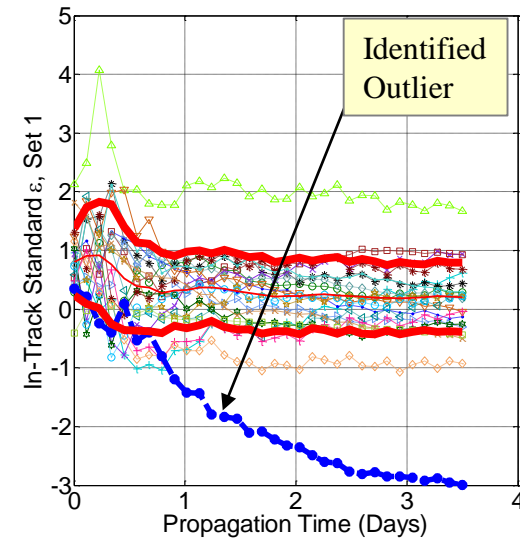**Step 7:** Tune Covariance if the Pass Percentage falls under a User Specified Threshold

- The acceleration variances in Step 1 can only be changed after the tuning process. Updated variances are configuration managed and require approval before they are deployed to operations.

- Step 2 is performed as part of the nominal daily product delivery.

- Steps 3 to 6 represent the QA of the covariance and are performed via automation using FreeFlyer and MATLAB.

- QA of Aqua and Aura covariances is performed over a rolling 90-day timespan.

- Testing with a 3-day cadence is statistically required in order to isolate the affects of the 2 ½ days worth of rolling Tracking and Data Relay Satellite (TDRS) observations that go into daily Flight Dynamics Facility (FDF) orbit determination runs.

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Covariance QA Automation
Visual Aids Presented to Analyst (1 of 2)

Example Aqua Set 1 QA Results for April 11, 2017 are given above.

Component Estimate Error plots give an idea of how far each component covariance is deviating from its mean root mean squared (RMS) component error.

An empirical 3-DOF Chi-Square distribution for each propagation point is assessed against its parent distribution.

The Cramer-Von Mises empirical distribution function (EDF) test is used to determine the likelihood each set of covariances represents a realistic distribution of the corresponding set of propagation errors tied to it – A "Pass Percentage" is used to determine Covariance Realism.

Covariance QA Automation
## Visual Aids Presented to Analyst (2 of 2)

- Standard Component Errors are available for Radial, In-Track, and Cross-Track directions. In-Track Standard Errors are utilized in Outlier Identification Process.

- Any propagations outside of the $\pm 1\sigma$ bounds in the In-Track Component are tested for outlier identification

- Normal Gaussian distribution based on Component Errors are also available.
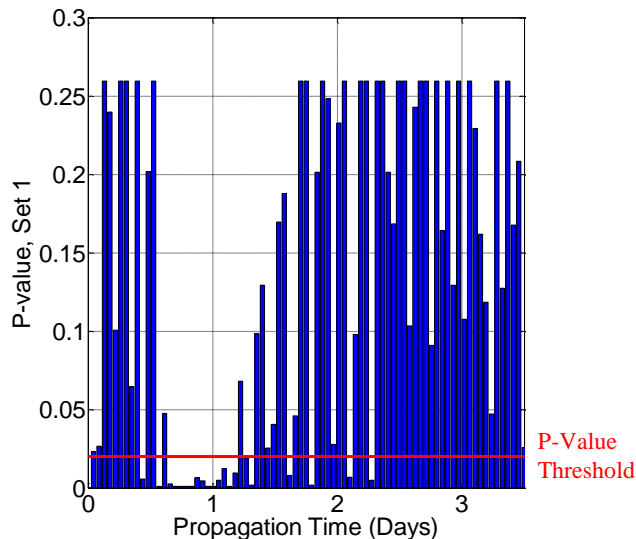
- The Probability Value (P-Value) vs. Propagation Time chart gives information regarding where in the propagation the covariances are passing the realism testing.

- A "Pass-Percentage" is calculated for all sets based on the P-values calculated through the timeframe at every step. Based on seasonal covariance tuning from 2014 to 2016, FDS recommended this threshold be set to 60% – a statistically commendable result.

- Periodicity in the Radial Propagation Error is causing low levels of realism between 0.5 to 1.25 days. The Covariance is oversized in this timeframe.

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Covariance QA Automation
Outlier Identification Confirmation


Potential Outlier Propagations
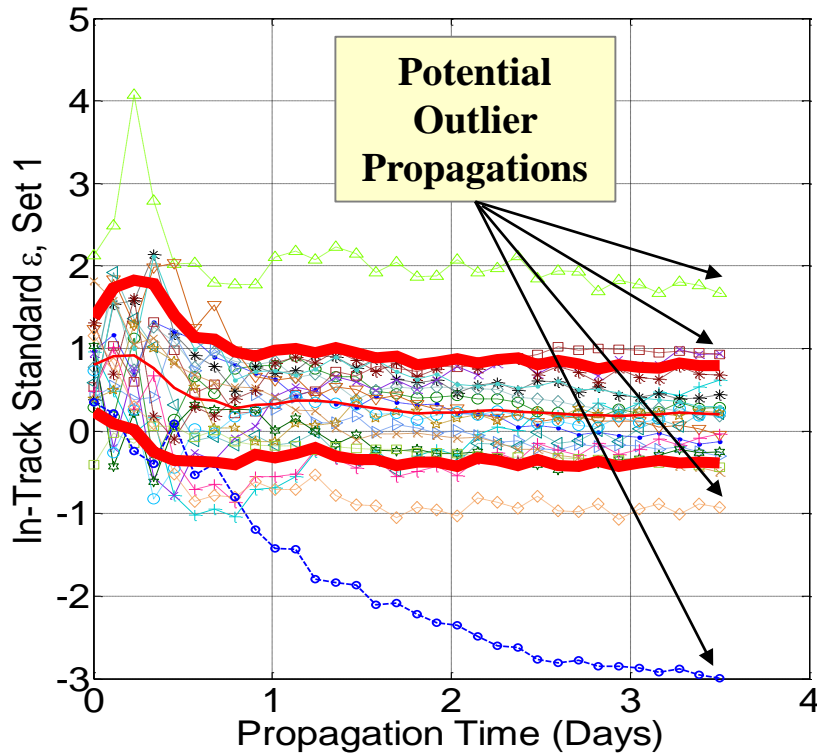
- Automation identifies potential outliers based on the In-Track standard errors. Propagations with an In-Track standard error outside $\pm 1\sigma$ bounds after 3.5 days will be tested.

- Automation uses a Rosner Outlier Test on any deviant normalized In-Track standard errors – the test will detect outliers that are either much smaller or larger than the rest of the data and is designed to avoid the problem of masking, where an outlier close to another outlier goes undetected.

- The outliers are entered into the test in order of most to least deviant.

- Naturally, the solar activity in the timeframe of the propagation start date is used to determine if there was a peak or persistently high solar activity. See figure to the left.

April 1st, 2017
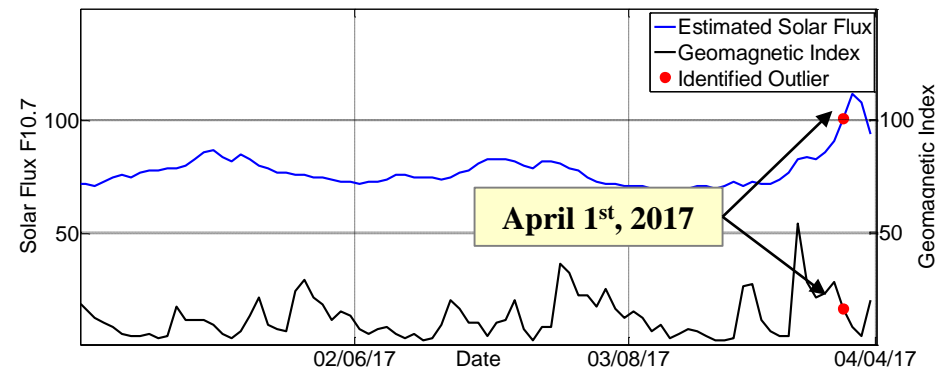
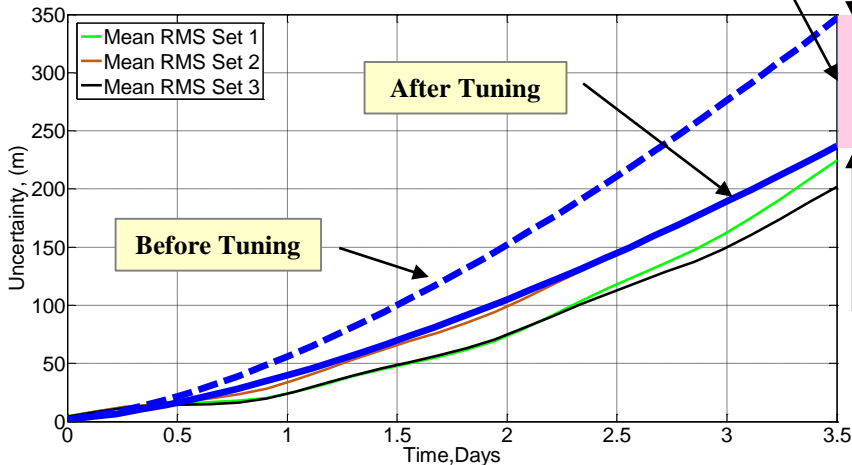**Note: Only the four most deviant propagations are tested using the Rosner Outlier Test.**

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Covariance QA Automation
Tune the Covariance (Aqua)

Radial Covariance vs. Mean Error from QA Sets

In-Track Covariance vs. Mean Error from QA Sets

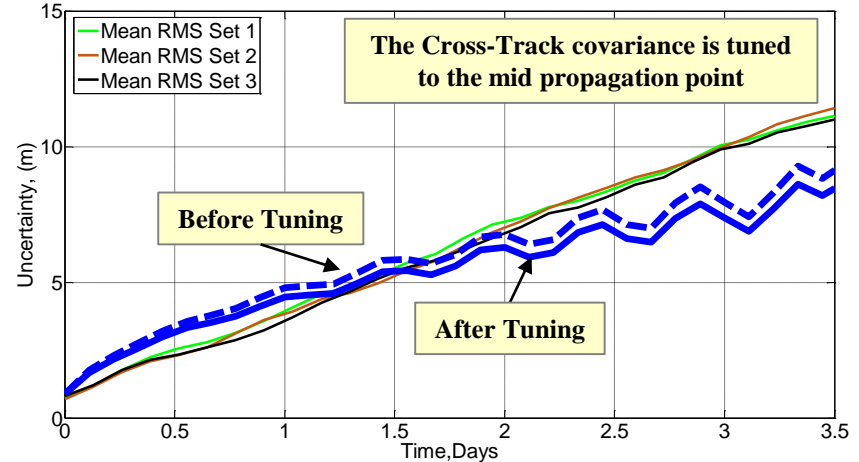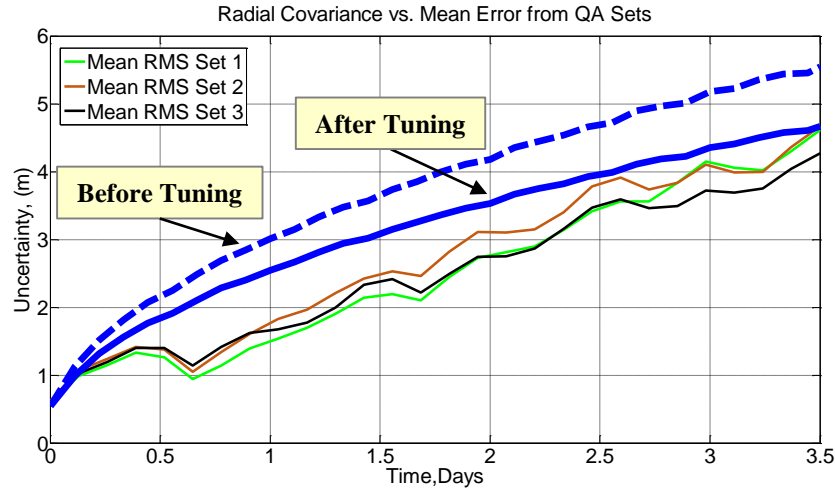Cross-Track Covariance vs. Mean Error from QA Sets
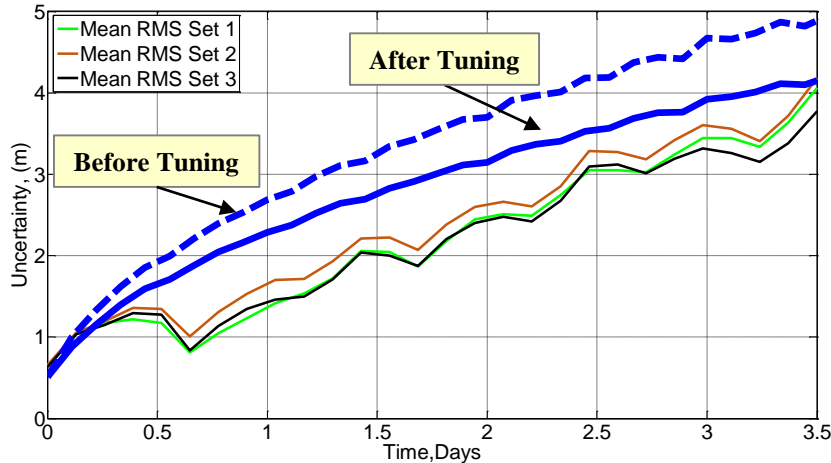
- Aqua's P-value Pass Percentage decreased below the FDS imposed threshold (60%) on November 7, 2016. Aqua was tuned to improve covariance realism.

- The component acceleration variances are changed until the Pass Percentages for all three sets of covariances exceed the user specified threshold.

- The current strategy is to tune the covariance to the **largest Mean RMS Component Error in the Radial and In-Track directions at the final propagation point and to the mid propagation point in the Cross-Track direction** (to achieve the highest level of realism).
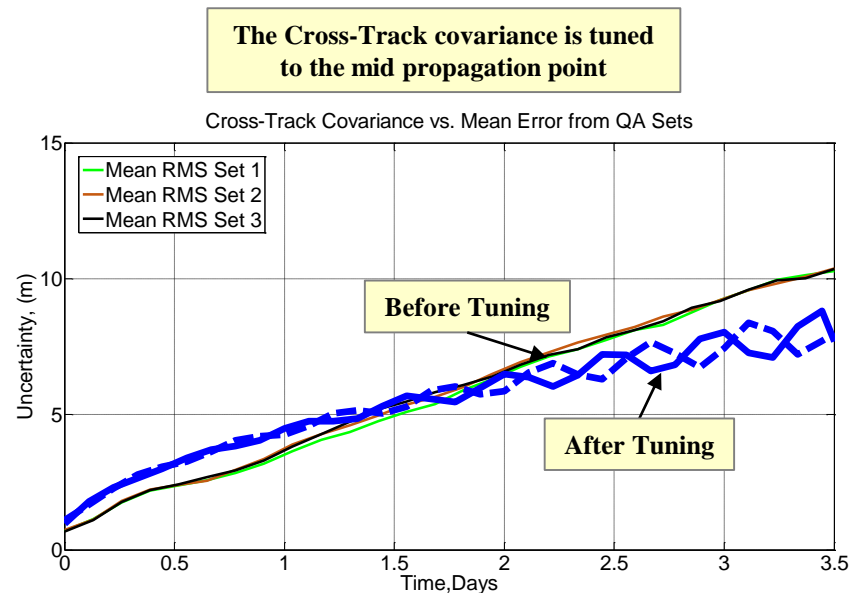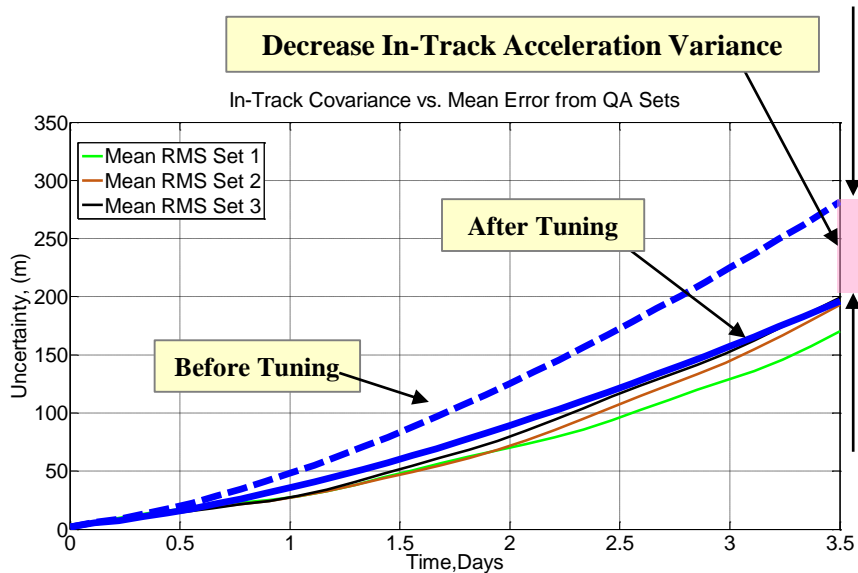
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Covariance QA Automation
Tune the Covariance (Aura)

Radial Covariance vs. Mean Error from QA Sets

- Mean RMS Set 1
- Mean RMS Set 2
- Mean RMS Set 3

After Tuning

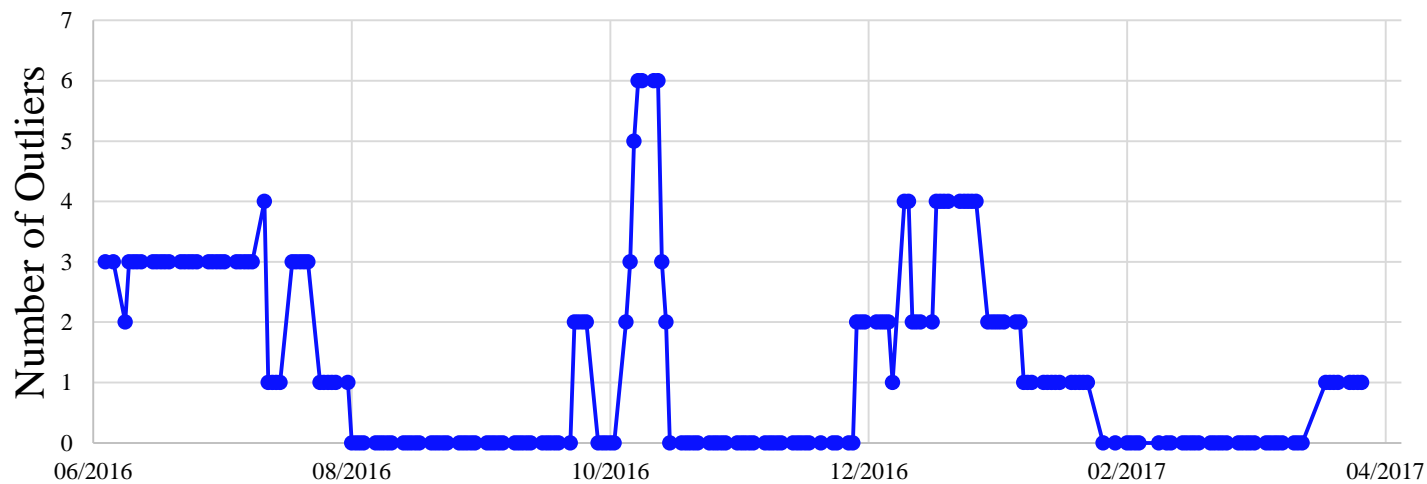Before Tuning

- Aura's Covariance was tuned in parallel with Aqua (on November 9, 2017). Aura's P-value Passing Percentage was improved after tuning and Aura did not fall under the FDS imposed threshold (60%).

- A similar tuning strategy was applied to Aura's covariance.

Decrease In-Track Acceleration Variance

In-Track Covariance vs. Mean Error from QA Sets

- Mean RMS Set 1
- Mean RMS Set 2
- Mean RMS Set 3

After Tuning

Before Tuning

The Cross-Track covariance is tuned to the mid propagation point

Cross-Track Covariance vs. Mean Error from QA Sets

- Mean RMS Set 1
- Mean RMS Set 2
- Mean RMS Set 3

Before Tuning

After Tuning

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Mission Operations Working Group
June 13-15, 2017
Automated Covariance QA Results to Date (Aqua)
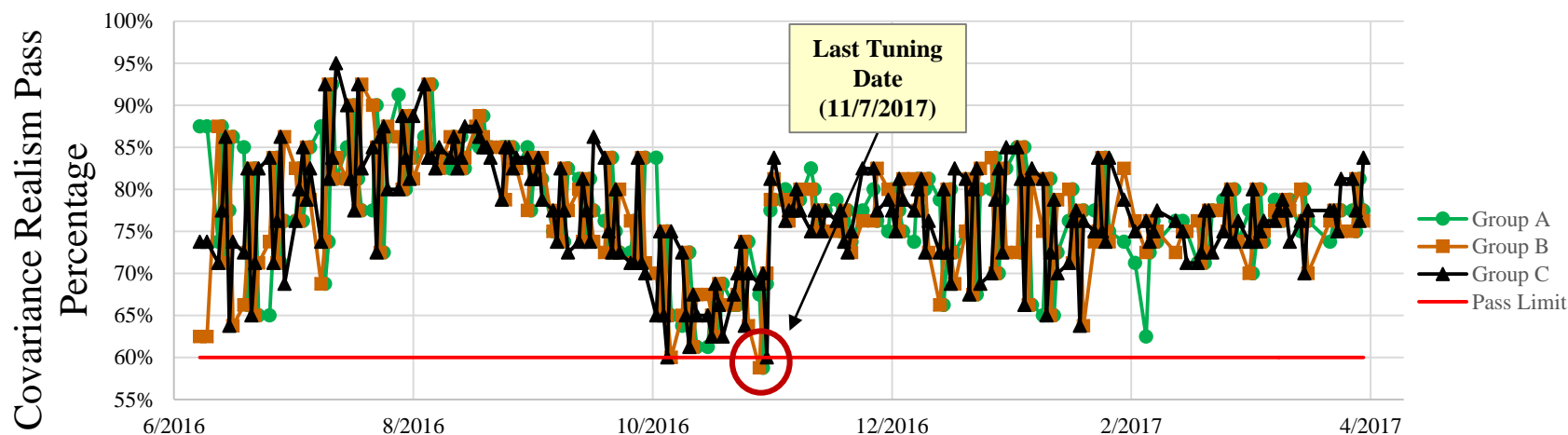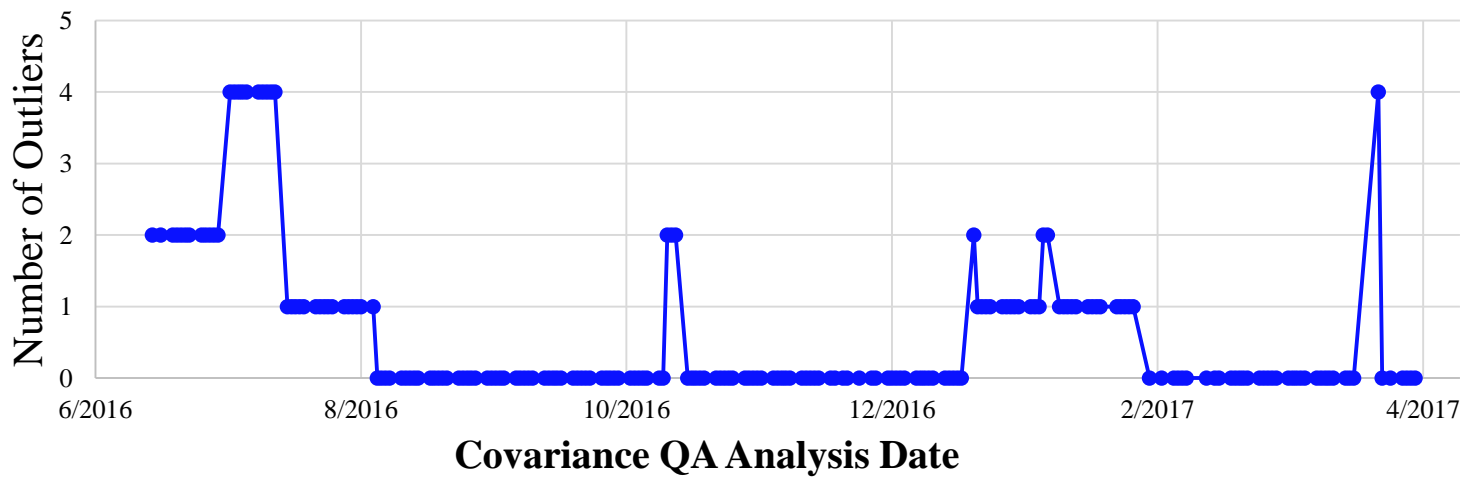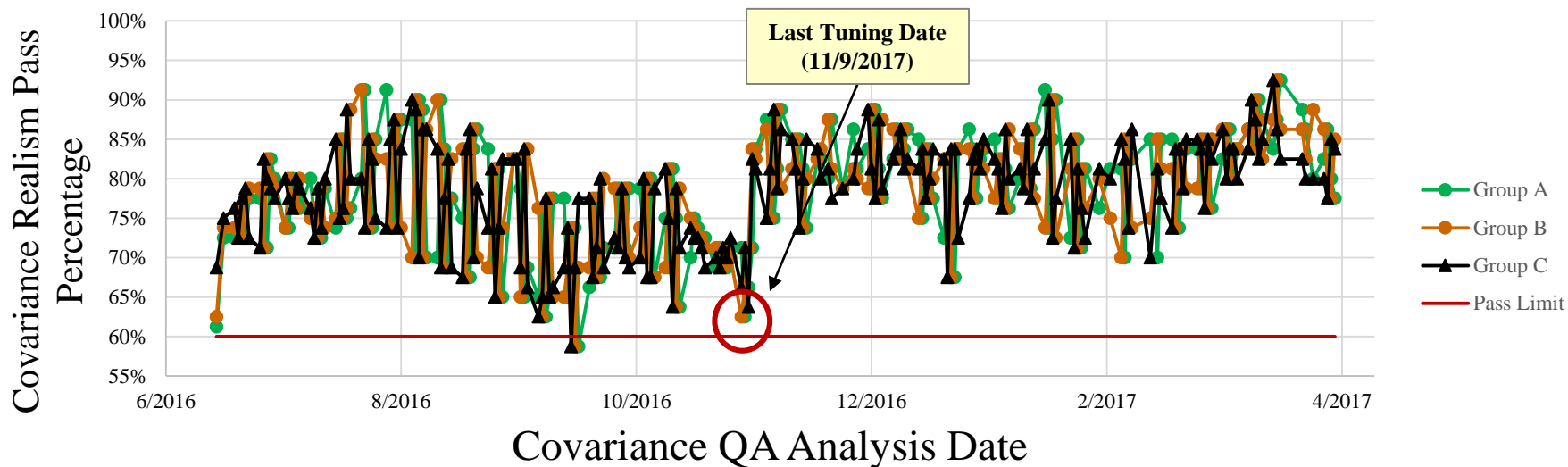

Covariance QA Analysis Date

Automated Covariance QA Results to Date (Aura)

Overview of Covariance Propagation through Maneuvers

- The method that EOS FDS has adapted to account for maneuver execution error includes the following stages:

  1. Gaussian distribution testing and outlier investigation of the Delta-V component errors **Operational**
  2. Creation of a Error Covariance Matrix using the preceding Delta-V component errors **Operational**
  3. Error Covariance Matrix Scaling using RIC Scale Factors **Operational**
  4. Application of Dataset Biases (An Enhancement to the Maneuver Planning Process) **Future**

- The preceding method is tested by propagating the Error Covariance Matrix through historical maneuvers and performing a covariance realism analysis on the resulting predicted post-maneuver propagation errors.

- The mean of each of the Radial, In-Track, and Cross-Track ΔV components constitutes the maneuver execution bias, μ.

- There are two approaches to make use of bias:

  EOS FDS Preferred Approach

  Approach 1 – Add bias to the planned maneuver plan. For example, if an In-Track ΔV component of 10 cm/s is planned and the bias is +2.5 cm/s then this bias is added to the satellite's state estimate propagation at the time of the maneuver.

  Approach 2 – Add bias to the commanded maneuver plan. For example, if an In-Track ΔV component of 10 cm/s is planned and the bias is +2.5 cm/s then the plan is changed to 7.5 cm/s prior to the propagation of the satellite's state estimate.

Covariance Propagation through Maneuvers
Future Work

- EOS FDS is investigating new methods of adding the maneuver Execution Error Sample Covariance to the propagated covariance throughout inclination adjust maneuvers (IAMs).

- Updated Linearized Covariance Propagation – The formula for linearly propagating covariance through maneuvers:

$$P(t_n) = \Phi(t_n, t_{n-1})^T (P(t_{n-1}) + Q_m(t))P(t_{n-1})\Phi^T(t_n, t_{n-1}) + Q(t)$$

  where $P(t_n)$ = Initial Covariance Matrix

  $\Phi(t_n, t_{n-1})$ = State Transition Matrix

  $Q(t)$ = Process Noise Matrix

  $Q_m(t)$ is non-zero only during the execution of the maneuver

- This method will be analyzed for Aqua and Aura DMUs for improved covariance realism. IAMs will be an extended case of DMUs.

Future Work
Polynomial Chaos Expansion (PCE)

- EOS FDS is exploring a new method of covariance propagation by using Polynomial Chaos Expansion (PCE) methods. This method is based on the paper *Conjunction Assessment Using Polynomial Chaos Expansions* by Brandon Jones, Alireza Doostan, and George Born, in which PCEs were used to calculate conjunction P_c.

- PCE Methods maps stochastic inputs (in this case, some initial position/velocity state) to a spectral polynomial solution space. That is to say, a spacecraft state can be approximated by:

$$\hat{X}(t,\xi) = \sum_{\alpha \in \Lambda_{p,d}} c_\alpha(t)\psi_\alpha(\xi)$$

where X is the position/velocity state, ξ is the stochastic input, ψ_α is the basis polynomial being mapped to (in this case, Hermite Polynomials), and c_α is the coefficient of the polynomial (to be solved).
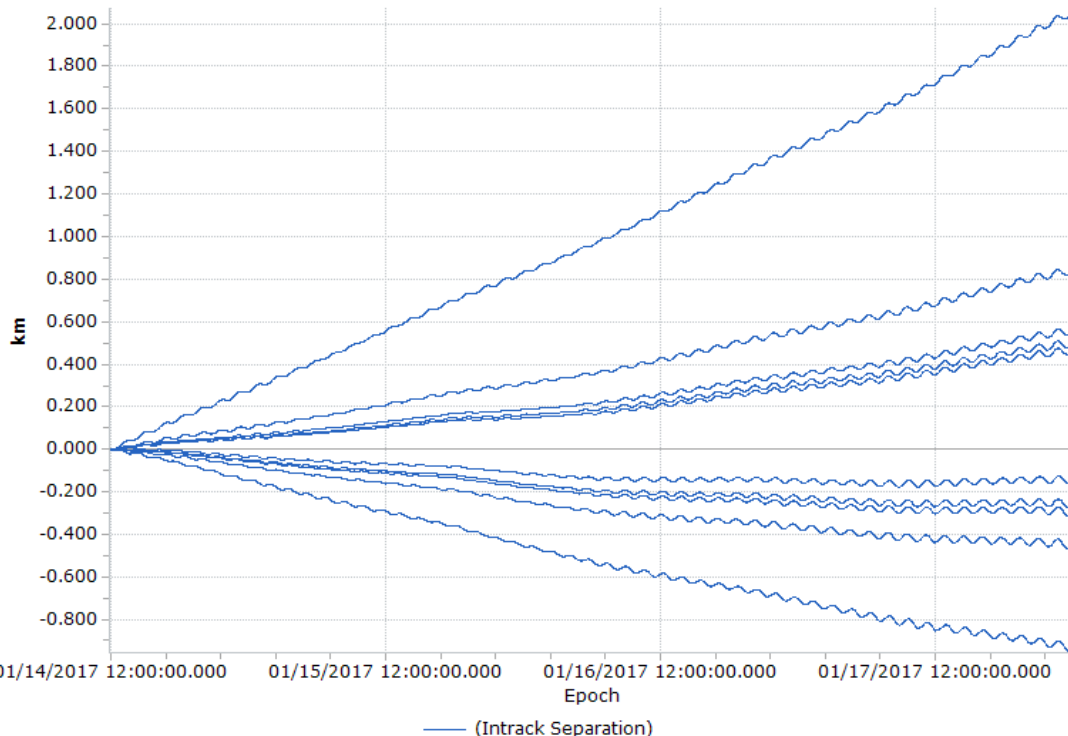
Polynomial Chaos Expansion (PCE)
Method of Solving

- PCE Equations are solved for by mapping multiple stochastic inputs to corresponding outputs of the desired model. In this case, multiple propagations of the Aqua spacecraft based on a Gaussian distributed initial states (such as graphed below).

**Intrack Xi Realization/Definitive Separation**
04/25/2017

Method:
1. Generate N realizations (based on the number of coefficients to solve for) of ξ_i which are Guassian distributed.
2. For each ξ_i, use initial $X(t, \xi)$ based on the random input ξ_i and propagate $X(t, \xi)$ to some time t for each N realizations (graphed on the left).
3. Solve for $c_\alpha(t)$ based on the N final states $X(t_f, \xi_f)$ (in this case, by using least-square regression)

Polynomial Chaos Expansion (PCE)
Advantages

- PCEs can be very useful when Gaussian uncertainties to the model input are introduced (such as possibly during high solar activity, maneuvers, and spacecraft configuration changes).

- Once the coefficients of the PCE are solved, it is a complete state representation of the system. Thus, one could use the PCE approximations in Monte Carlo type analysis where propagations could instead be replaced by evaluations of PCE Polynomials—a much less computationally demanding method.

- There is enough generalizations in mapping input that it can be applied to more than just position/velocity state variations. Could be applied to other type of inputs, such as yaw angle and burn time uncertainties during maneuvers.

- This may help improve low periods of realism during 0.5-1.25 days of propagation, see right.

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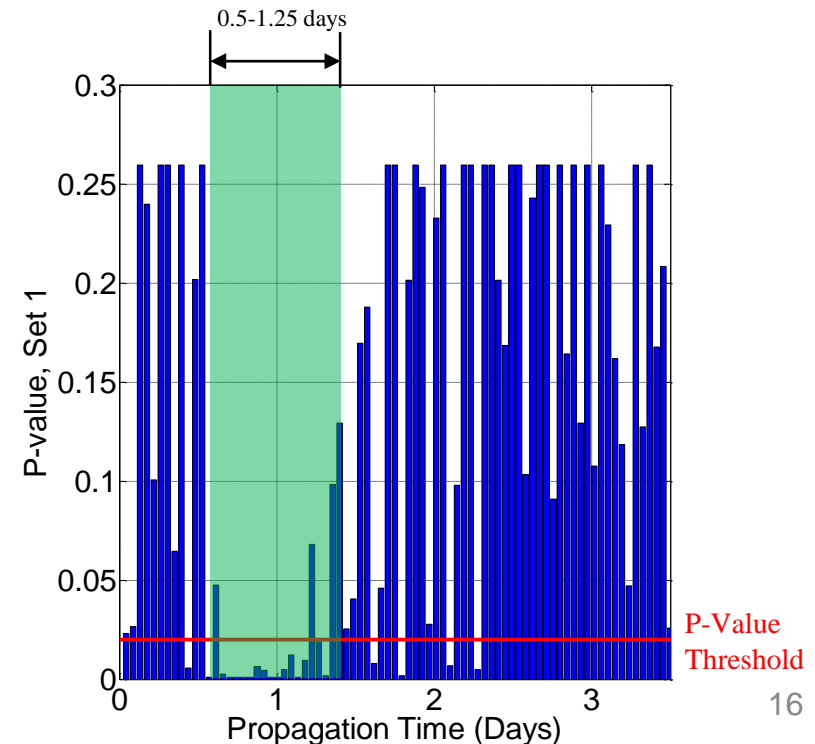
- The EOS FDS team has been using tuned O/O covariance for Aqua and Aura:
  - The automation of the covariance QA has been established, tested, and working as expected.
  - The automation of outlier identification has been established, tested, and working as expected.
- The EOS FDS team has been using O/O covariance with maneuver execution error assuming zero-bias for DMUs:
  - Gaussian distribution testing of the maneuver component errors has been established and working as expected.
  - Maneuver Execution Error Covariances to be updated on a bi-annual basis.
- The EOS FDS team is looking into new methods of covariance propagation throughout maneuvers and new covariance propagation methods using Polynomial Chaos Expansion (PCE).